GROUP THEORY

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Abstract:
Group theory is the study of algebraic structures called groups. It is an important part in present day mathematics, was established early in the nineteenth century in connection with the solutions for algebraic equations. Originally, a group was the set of all permutations of the roots of an algebraic equation which has the property that the combination of any two of these permutations again belongs to the set. Later the idea was generalized to the concept of an abstract group. An abstract group is essentially the study of a set with an operation defined on it. Group theory has many useful applications both within and outside mathematics. Groups arise in a number of apparently unconnected subjects. In fact, they appear in crystallography and quantum mechanics, in geometry and topology, in analysis and algebra, and even in biology. Before we start talking about a group it is beneficial to discuss the binary operation on a set, because these are sets on whose elements algebraic operations can be made. We can obtain a third element of the set by combining two elements of a set. It is not always true, which is why this concept needs attention.

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Keywords:
C is set of complex numbers.
R is set of real numbers.
Q is set of rational numbers.
Z is set of integers.
N is set of natural numbers.

Set:
Set is well defined collection of distinct objects. For example

{tiger, lion, puma, cheetah, leopard, cougar, ocelot}

is a set of large species of cats.

Semigroup:
If S is a non empty and * be a binary operation on S, then the algebraic system {S,*} is called a semigroup, if the operation * is associative i.e. if for any a, b, c €S then

(a*b)*c=a*(b*c)

For example, if S is the set of positive even numbers, then {E, +} is a semigroup.

Monoid:
If a semigroup {M,*} has an identity element with respect to the operation *, then {M,*} is called a monoid i.e. if

- for any a, b, c € M, (a*b)*c=a*(b*c)
- if there exists an element e€M such that for any a€M, a*e=e*a

then the algebraic system {M,*} is called a monoid.
For example, if N is the set of natural numbers, then {N, X} is monoid with the identity 1.

Group:
A group is a monoid with an inverse element. The inverse element denoted by I of a set is an element such that (a*I)=(I*a)=a, for each element a€G.
So,a group holds four properties,

- Closure,